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Adaptive Quaternion Feedback Regulation for Eigenaxis Rotations

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A controller to rotate a rigid body between two successive orientations is designed. Particular features are the fact that it is based on the quaternion approach, known to provide singularity-free attitude description, and it is adaptive in the sense that it does not need specific knowledge of the inertia matrix. Global stability of the overall controller is proved analytically and tested in computer simulations.

I. Introduction

THE problem of orienting a rigid body with respect to inertial coordinates is of considerable interest in the control of spacecraft and robotic manipulators.

A basic issue is how to express the rotation of the body-fixed reference frame with respect to a suitable inertial frame. It is well known that use of a minimal representation by the three Euler angles yields not only nonlinear kinematic equations but also singular orientations where the solution is undetermined. By the addition of a degree of redundancy, it has been shown by several authors that the problems with Euler angles can be avoided.

The method of quaternions, discussed in Refs. 1 and 2, dates back to Euler and Hamilton in the 1700s and 1800s. It is based on the fact, proved by Euler, that any rotation of a frame with respect to another can be represented by a rotation around a properly chosen vector (the eigenaxis). This is another way of saying that any rotation matrix, apart from the trivial case of identity, has a unique eigenvector corresponding to the eigenvalue at 1.[3]

Control systems based on the quaternion approach have been introduced by Mortensen[4] and Ickes.[5] For large maneuvers of spacecrafts Wie et al.[6,7] have shown global stability and robustness of the nonlinear controller. Optimal open- and closed-loop maneuvers are given in Ref. 8 for combined use of reaction wheels and thrusters. Spacecraft orientation is currently described in terms of quaternions for the Space Shuttle and Galileo.[9] A general framework for the attitude control problem for rigid bodies is given in Ref. 10.

A recent survey paper[11] presents properties and an extensive list of applications of the quaternion approach.

In space applications, the main parameters that need to be known to the designer are the parameters of the inertia matrix. According to the particular application, these can be changing with time due to several reasons, such as fuel consumption. The development of space robot concepts that envision mechanical manipulators performing such diverse operations as grabbing objects[12] calls for self-tuning control schemes. In the particular case of space robotics the inertia matrix changes with load and operating conditions.[12]

A previous approach by Slotine and DiBenedetto[13] focuses on the development of an adaptive controller based on the Gibbs representation suitable for rotations of angles smaller than 360 deg.

In this paper, we design a nonlinear adaptive controller that performs vehicle orientation in the presence of large uncertainties of the inertia matrix. The concept of adaptive control for linear time-invariant systems dates back to the 1950s although reliable designs of proven stability did not appear before the late 1970s.[14,15] Both direct and indirect adaptive control schemes are presented. The direct adaptive controller has the advantage of simplicity in the implementation, since it is based on a simple gradient-type adaptation law. However, the indirect approach we present, although of more complex implementation, is based on a recursive least-squares identification algorithm that has the advantage of converging faster.

In general, adaptive controllers of nonlinear systems are based on a linearized model of the plant, and stability can be shown only locally. In a few cases the particular structure of a nonlinear system lends itself to a globally stable adaptive control algorithm, such as the case of robotic manipulators.[16] The adaptive controller presented below falls in this category, for which we can show global stability of the overall system composed of the plant and the controller with recursive estimates of the inertia matrix.

Eigenaxis rotation is presented in Sec. II. Three control techniques are given next: a sliding-mode technique in Sec. III, direct adaptive control in Sec. IV followed by an indirect approach in Sec. V, whereas examples and conclusions are given in Sec. VI and VII, respectively.

II. Eigenaxis Rotation

The problem of rotating a rigid structure using a body-fixed torquing device has been widely treated in the literature. In particular, in Ref. 7 it is shown that using Euler's equations of motion, any change in orientation of a rigid body can be accomplished by a rotation around an axis called the *eigenaxis*. This defines the *quaternion* as a vector $[q_1, q_2, q_3, q_4]^T \in R^4$ such that

$$q_i = c_i \sin\left(\tfrac{1}{2}\phi\right) \qquad i = 1, 2, 3$$
$$q_4 = \cos\left(\tfrac{1}{2}\phi\right) \tag{1}$$

with $c = [c_1, c_2, c_3]^T$ being the unitary vector parallel to the eigenaxis in the reference frame and ϕ the angular rotation around the eigenaxis itself.

If we define $\boldsymbol{q} = [q_1, q_2, q_3]^T$, it is shown in Ref. 7 that, given the angular velocity vector $\omega = [\omega_1, \omega_2, \omega_3]^T$ the quaternion satisfies the differential equations

$$\dot{\boldsymbol{q}} = \tfrac{1}{2}\Omega \boldsymbol{q} + \tfrac{1}{2}q_4\omega$$
$$\dot{q}_4 = -\tfrac{1}{2}\omega^T \boldsymbol{q} \tag{2}$$

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with $\boldsymbol{\Omega}$ being the skew symmetric matrix

$$\boldsymbol{\Omega} = -\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3}$$

It is well known that the quaternion vector $[\boldsymbol{q}^T, q_4]^T$ as defined in Eq. (1) has the property that its magnitude is constant for all $t > 0$, i.e.,

$$\boldsymbol{q}^T(t)\boldsymbol{q}(t) + q_4(t)^2 = \boldsymbol{q}^T(0)\boldsymbol{q}(0) + q_4(0)^2 = 1 \tag{4}$$

The angular velocity vector $\boldsymbol{\omega}$ itself is the solution of the differential equation of motion

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = \boldsymbol{\Omega}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{f} + \boldsymbol{u} \tag{5}$$

with $\boldsymbol{J}$ being the inertia matrix and $\boldsymbol{u} = [u_1, u_2, u_3]^T$ the control torque vector. The term $\boldsymbol{f}$ indicates dynamic errors in the model, due to viscous friction or other effects in general due to the interaction between the system to be controlled and its environment. In this research we assume that this perturbation $\boldsymbol{f} = [f_1, f_2, f_3]^T$ is unknown and that rough estimates of its bound are available,

$$|f_i(t)| \le F_i(\boldsymbol{\omega}, \boldsymbol{q}, t), \qquad i = 1, 2, 3 \tag{6}$$

with F_i satistying the assumption

$$\boldsymbol{\omega} \in L_\infty \Rightarrow \boldsymbol{F} \in L_\infty \tag{7}$$

(With L_∞ we denote the set of uniformly bounded functions in R.) Given the two equations for motion [Eq. (5)] and quaternion [Eq. (2)], stable control laws have been designed[6,7] in order to take the rigid body (a spacecraft in particular) from an initial angular position to a desired one, provided the inertia matrix $\boldsymbol{J}$ is known. In the particular case of exact modeling (when the uncertainty term $\boldsymbol{f} = \boldsymbol{0}$), it is shown that a control law of the form

$$\boldsymbol{u} = -\boldsymbol{\Omega}\boldsymbol{J}\boldsymbol{\omega} - \boldsymbol{D}\boldsymbol{\omega} - \boldsymbol{K}\boldsymbol{q} \tag{8}$$

with properly chosen gains $\boldsymbol{D}$ and $\boldsymbol{K}$ yields stable regulation, in the sense that the quaternion vector tends to $[\boldsymbol{q}(\infty), q_4(\infty)] = [0, 0, 0, 1]$, causing the body to align with the reference frame starting from any orientation.

III. Sliding-Mode Control

When the moment of inertia $\boldsymbol{J}$ is known at least within a certain approximation, we are able to design a controller to perform regulation in the presence of model uncertainties $\boldsymbol{f}$ with known bounds $\boldsymbol{F}$. Using an approach well known in the literature, we identify a class of *sliding surfaces* in the state space $[\boldsymbol{\omega}, \boldsymbol{q}]$ in R^6 along the same lines of Ref. 17. In particular we show in the following that, for any positive constant, α, the surface

$$\boldsymbol{\omega} + \alpha\boldsymbol{q} = \boldsymbol{0} \tag{9}$$

is a sliding surface in the sense that $\boldsymbol{\omega} + \alpha\boldsymbol{q} \to 0$ implies $\boldsymbol{\omega}, \boldsymbol{q} \to 0$ as $t \to \infty$. In particular we can show the following.

Theorem 1. Let α be an arbitrary positive constant and define

$$\boldsymbol{e} = \boldsymbol{\omega} + \alpha\boldsymbol{q} \tag{10}$$

If $\dot{\boldsymbol{\omega}}, \boldsymbol{\omega} \in L_\infty$ and

$$\lim_{t\to\infty} \boldsymbol{e}(t) = \boldsymbol{0}$$

then

$$\lim_{t\to\infty} \boldsymbol{\omega}(t) = \boldsymbol{0}, \qquad \lim_{t\to\infty} \boldsymbol{q}(t) = \boldsymbol{0}$$

To prove this theorem, we can make use of the interesting result shown by Vadali[18] that shows that the sliding surface (9) is optimal in the sense that it minimizes the cost function

$$V = \int_0^\infty \alpha^2\boldsymbol{q}^T\boldsymbol{q} + \boldsymbol{\omega}^T\boldsymbol{\omega}\,\mathrm{d}t$$

which implies that both $\boldsymbol{q}$ and $\boldsymbol{\omega}$ are L_2 functions. *Most of the time* this implies that $\boldsymbol{\omega}, \boldsymbol{q} \to 0$ apart from some particular cases of signals with unbounded derivatives. A technical and rigorous proof of this theorem is given in Ref. 19.

A sliding-mode regulator can be designed to compensate for the dynamic uncertainty $\boldsymbol{f}$. In particular, let the control input be

$$\boldsymbol{u}(t) = -\gamma\boldsymbol{J}\boldsymbol{\omega}(t) - \gamma\alpha\boldsymbol{J}\boldsymbol{q}(t) - \boldsymbol{\Omega}(t)\boldsymbol{J}\boldsymbol{\omega}(t) - \alpha\boldsymbol{J}\dot{\boldsymbol{q}}(t) + \bar{\boldsymbol{u}}(t)$$

with $\bar{\boldsymbol{u}}(t) = [\bar{u}_1(t), \bar{u}_2(t), \bar{u}_3(t)]$, $\quad i = 1, 2, 3$, and

$$\bar{u}_i(t) = -F_i(\boldsymbol{\omega}, \boldsymbol{q}, t)\,\mathrm{sgn}[e_i(t)]$$

Then we can show that the state $\boldsymbol{\omega}, \boldsymbol{q}$ of the closed-loop system tends to the surface $\boldsymbol{\omega} + \alpha\boldsymbol{q} = 0$ and *slides* to the equilibrium point $[\boldsymbol{\omega}, \boldsymbol{q}] = \boldsymbol{0}$. This can be easily seen by combining the dynamic equation (5) with the control input to obtain

$$\boldsymbol{J}(\dot{\boldsymbol{e}} + \gamma\boldsymbol{e}) = \bar{\boldsymbol{u}} + \boldsymbol{f}$$

Definition of the Lyapunov function

$$V(\boldsymbol{e}) = \tfrac{1}{2}\boldsymbol{e}^T\boldsymbol{J}\boldsymbol{e}$$

with derivative

$$\dot{V}(\boldsymbol{e}) = -\gamma\boldsymbol{e}^T\boldsymbol{J}\boldsymbol{e} + \sum_{i=1}^{3} e_i(t)[\bar{u}_i(t) + f_i(t)] \le 0$$

proves the result.

IV. Direct Adaptive Control

In the adaptive case, we assume the inertia matrix $\boldsymbol{J}$ to be unknown to the designer or changing under different load conditions, and therefore we replace the control input in Eq. (8) by the expression

$$\boldsymbol{u}(t) = -\gamma\hat{\boldsymbol{J}}(t)\boldsymbol{\omega}(t) - \gamma\alpha\hat{\boldsymbol{J}}(t)\boldsymbol{q}(t) - \boldsymbol{\Omega}(t)\hat{\boldsymbol{J}}(t)\boldsymbol{\omega}(t) - \alpha\hat{\boldsymbol{J}}(t)\dot{\boldsymbol{q}}(t) + \bar{\boldsymbol{u}}(t) \tag{11}$$

with α and γ being arbitrary positive constants. The term $\hat{\boldsymbol{J}}(t)$ represents the estimate of the inertia matrix $\boldsymbol{J}$ at time $t \in R$. Notice that the term $\dot{\boldsymbol{q}}$ in the control input can be computed from Eq. (2) without involving differentiators.

In the sequel we determine an algorithm to recursively update the estimates $\hat{\boldsymbol{J}}(t)$ of the inertia matrix and compensate for the uncertainties $\boldsymbol{f}$ based on a suitable error signal. In order to make the argument more fluent, first define the following signals:

$$\boldsymbol{\theta} = [J_{11}, J_{12}, J_{13}, J_{22}, J_{23}, J_{33}]^T, \qquad \boldsymbol{e}(t) = \boldsymbol{\omega}(t) + \alpha\boldsymbol{q}(t) \tag{12}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \epsilon_1 & \epsilon_2 - \omega_1\omega_3 & \epsilon_3 + \omega_1\omega_2 & -\omega_2\omega_3 & \omega_2^2 - \omega_3^2 & \omega_2\omega_3 \\ \omega_1\omega_3 & \epsilon_1 + \omega_2\omega_3 & \omega_3^2 - \omega_1^2 & \epsilon_2 & \epsilon_3 - \omega_1\omega_2 & -\omega_1\omega_3 \\ -\omega_1\omega_2 & \omega_1^2 - \omega_2^2 & \epsilon_1 - \omega_2\omega_3 & \omega_1\omega_2 & \epsilon_2 + \omega_1\omega_3 & \epsilon_3 \end{bmatrix}^T \tag{13}$$

with $\boldsymbol{\epsilon}(t) = [\epsilon_1, \epsilon_2, \epsilon_3]$ defined as

$$\boldsymbol{\epsilon}(t) = \gamma\boldsymbol{\omega}(t) + \alpha\gamma\boldsymbol{q}(t) + \alpha\dot{\boldsymbol{q}}(t) \tag{14}$$

Then we show the following.

Theorem 2. The plant defined by Eqs. (5) and (2) and control input (11) is globally asymptotically stable and

$$\lim_{t\to\infty}\left[\boldsymbol{q}^T(t), q_4(t)\right] = [0, 0, 0, 1] \tag{15}$$

provided the inertia matrix estimate $\hat{\boldsymbol{J}}(t)$ is updated as

$$\dot{\hat{\boldsymbol{\theta}}}(t) = \lambda\boldsymbol{\Phi}(t)\boldsymbol{e}(t) \tag{16}$$

and the input term $\bar{u} = [\bar{u}_1, \bar{u}_2, \bar{u}_3]^T$ is given by

$$\bar{u}_i(t) = -F_i(\omega, q, t)\text{sgn}[e_i(t)], \qquad i = 1, 2, 3 \tag{17}$$

where λ is an arbitrary positive constant.

The proof follows the derivation of preliminary results.

In order to show the global stability and tracking properties of the adaptive system, let us write the control input (11) as

$$u(t) = -\gamma J\omega(t) - \gamma\alpha Jq(t) - \Omega J\omega(t) - \alpha J\dot{q}(t) + \tilde{u}(t) + \bar{u}(t) \tag{18}$$

with

$$\tilde{u}(t) = \gamma\tilde{J}(t)\omega(t) + \gamma\alpha\tilde{J}(t)q(t) + \Omega\tilde{J}(t)\omega(t) + \alpha\tilde{J}(t)\dot{q}(t) \tag{19}$$

$$\tilde{J}(t) = J - \hat{J}(t) \tag{20}$$

It is just a matter of algebra to write Eq. (19) in a compact form:

$$\tilde{u}(t) = \Phi^T(t)\tilde{\theta}(t) \tag{21}$$

where we define the parameter error vector

$$\tilde{\theta}^T = [\tilde{J}_{11}, \tilde{J}_{12}, \tilde{J}_{13}, \tilde{J}_{22}, \tilde{J}_{23}, \tilde{J}_{33}]$$

Combining (18) with (5), we obtain the dynamics of the system in terms of the parameter error vector

$$J[\dot{e}(t) + \gamma e(t)] = \tilde{u}(t) + f(t) + \bar{u}(t) \tag{22}$$

where the dynamic tracking error e is defined in Eq. (10).

Now we can show that the error signals e, $\tilde{\theta}$ and the angular velocity vector ω have the following properties.

Lemma 1. The system defined by Eqs. (22), (21), (16), and (17) with arbitrary initial conditions is such that

a) $e \in L_2 \cap L_\infty$,
b) $\tilde{\theta} \in L_\infty$,
c) $\omega \in L_\infty$, and $\dot{\omega} \in L_\infty$, and
d) $\lim_{t\to\infty} e(t) = 0$,

with L_2 and L_∞ denoting the square-integrable and uniformly bounded functions, respectively.

Proof. Define the Lyapunov function

$$V(e, \tilde{\theta}) = \tfrac{1}{2}e^T(t)Je(t) + \tfrac{1}{2}(\lambda^{-1})\tilde{\theta}^T(t)\tilde{\theta}(t) \tag{23}$$

Then

$$\dot{V}(e, \tilde{\theta}) = \dot{e}^T Je + \lambda^{-1}\dot{\tilde{\theta}}^T\tilde{\theta}$$

Substituting for $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$ and using Eq. (16), we obtain

$$\dot{V}(e, \tilde{\theta}) = \dot{e}^T Je - e^T\Phi^T\tilde{\theta}$$

which, combined with Eqs. (22) and (21), yields

$$\dot{V}(e, \tilde{\theta}) = -\gamma e^T Je + e^T f + e^T\bar{u}$$

By definition of $\bar{u}$ it is easy to verify that the two rightmost terms of the above expression can be bounded as

$$e^T(t)\bar{u}(t) + e^T(t)f(t) = \sum_{i=1}^{3}[-|e_i(t)|F_i(t) + e_i(t)f_i(t)] \le 0$$

nonpositive since $0 \le |f_i| \le F_i$ by assumption. Therefore, substituting for $\Phi^T\tilde{\theta} = \tilde{u}$ and using (22) we obtain

$$\dot{V}(e, \tilde{\theta}) \le -\gamma e^T Je \le 0 \qquad \forall t \ge 0 \tag{24}$$

This implies $0 \le V(\infty) \le V(t) \le V(0) < \infty$ for all $t \ge 0$, and therefore

$$V(\infty) - V(0) = \int_0^{+\infty} \dot{V}(t)\,dt \le -\gamma\int_0^{+\infty} e^T(t)Je(t)\,dt \tag{25}$$

Now we make the following claims:

i) $e \in L_\infty$ and $\tilde{\theta} \in L_\infty$ since $0 \le V(t) \le V(0)$ for all $t \ge 0$;
ii) $e \in L_2$ from Eq. (25) and the fact that J is positive definite;
iii) $\omega \in L_\infty$ since $\omega = e - \alpha q$ from Eq. (10) and the fact that the vector q is uniformly bounded;
iv) since ω and $\tilde{\theta}$ are uniformly bounded, then in turn also Φ, f (by assumption), and u are uniformly bounded [this together with Eq. (5) shows that $\dot{\omega} \in L_\infty$]; and
v) from Eqs. (22) and (21) we can write

$$\dot{e} = -\gamma e + J^{-1}\Phi^T(t)\tilde{\theta}(t) + J^{-1}(f + \bar{u})$$

Since $\tilde{\theta}$, Φ, and e are bounded from i), ii), and iii) above, it follows that $\dot{e}$ is also bounded. Therefore, by the fact that $e \in L_2$ and from Barbalat's theorem,[17] it follows that $e \to 0$, which proves claim iv).

Now we have to show that from the dynamic error e being as in Lemma 1, the system with dynamics (5) and (2) yields $\omega \to 0$, $q \to 0$, i.e., asymptotic positioning of the rigid body along the inertial reference axis. This is easily done by applying the result of Theorem 1 in the previous section, for which $e \to 0$ and $\dot{\omega}, \omega \in L_\infty$ yield $\omega, q \to 0$.

V. Indirect Adaptive Control

From a control perspective, one of the advantages of using the quaternion approach is its optimality in terms of minimal angular rotation. It is shown in Ref. 7 that a control of the form (8) yields only one angular rotation around the eigenaxis. This fact holds provided the inertia matrix is known to the designer.

The adaptive controller presented in the previous section does not require explicit convergence of the inertia matrix. It is well known that a steepest descent algorithm like in Eq. (16) has far slower convergence than more efficient algorithms such as the recursive least squares (RLS). The price paid for faster convergence is the requirement for higher complexity.

In this section we present an adaptive controller based on RLS estimation of the inertia matrix parameters. For simplicity we *assume* that there is no dynamic perturbation, i.e., $f = 0$. In the general case when dynamic uncertainties f are present with known bounds F, the use of dead zones as in Ref. 20 would ensure the stability of the algorithm.

The overall structure of the proposed controller plus estimator is *hybrid* (or *two time scale*) in the sense that estimator and controller operate at two different updating rates. For analysis purposes we will keep assuming the controller to be operating in the analog domain, whereas the parameter estimator is updated at discrete times. Hybrid adaptive controllers have appeared in the literature[20–22] as a viable alternative to the all-digital and all-analog approaches. They all show that the adaptive gains can be updated at a lower rate and still guarantee global stability, at least in principle.

The error model for the adaptive controller is derived from Eq. (5), rewritten here for convenience,

$$u = -\Omega J\omega + J\dot{\omega} \tag{26}$$

linear in the parameter J. In order not to require differentiation of the angular velocity ω, we can filter both sides of (26) to obtain

$$u_f = -(\Omega J\omega)_f + J(\dot{\omega})_f \tag{27}$$

with the subscript f denoting filtering. By simple algebraic manipulations we can write

$$\mathbf{u}_f = \Phi_f^T\theta \tag{28}$$

with $\sigma > 0$ an arbitrary constant,

$$\dot{\Phi}_f(t) = -\sigma\Phi_f(t) + \sigma\Phi(t) \tag{29}$$

$$\dot{u}_f(t) = -\sigma u_f(t) + \sigma u(t) \tag{30}$$

and Φ being as in (13) with $\epsilon = \dot{\omega}$

Since Eq. (30) is valid for all $t \in R$ it also holds at the sampling instants as

$$u_f(t_k) = \Phi_f^T(t_k)\theta \tag{31}$$

By applying the RLS algorithm the estimate of θ is updated recursively as

$$\hat{\theta}_{k+1} = \hat{\theta}_k + P_{k+1}\Phi_f(t_k)\left[u_f(t_k) - \Phi_f^T(t_k)\hat{\theta}_k\right] \tag{32}$$

$$P_{k+1} = P_k - P_k\Phi_f(t_k)\left[I + \Phi_f^T(t_k)P_k\Phi_f(t_k)\right]^{-1} \times \Phi_f^T(t_k)P_k + \mathbf{Q} \tag{33}$$

with the nonnegative definite matrix $\mathbf{Q}$ representing a forgetting factor. Well-known properties of the RLS algorithm are shown by different authors. In particular, it is shown in Ref. 23 that a limit always exists as

$$\lim_{k\to\infty} \hat{\theta}_k = \hat{\theta}_\infty \tag{34}$$

which is not necessarily the actual value of the parameters unless the matrix $\Phi_f(t_k)$ is persistently exciting (PE), i.e.,

$$\lim_{N\to\infty} \lambda_{\min}\left[\sum_{k=0}^{N} \Phi_f(t_k)\Phi_f^T(t_k)\right] = \infty \tag{35}$$

with $\lambda_{\min}$ denoting the minimum eigenvalue. Although this condition is difficult to verify analytically, extensive simulations show that the transient signals during regulation are sufficiently exciting to drive the parameter estimates to their correct values.

Once an estimate of the inertia matrix parameters is obtained, any of the control techniques designed for the known inertia matrix can be applied substituting its true value J with the current estimate $\hat{J}(t)$. Simulation results shown in the next section point to the fact that in general $\hat{J}(t) \to J$ very rapidly, thus providing a satisfactory control action.

VI. Simulation Results

In this section we present two simulation results pertinent to the direct and indirect adaptive algorithms presented in Secs. IV and V, respectively.

In the first example we simulate an extreme case when the inertia matrix is completely unknown, and the system performs the desired regulation with the direct adaptive controller. The second example is an indirect adaptive controller for a crew/equipment retriever (CER).[12] This controller is designed to perform optimally (in the sense of minimal angular rotation) when the inertia matrix is uncertain.

A. Direct Adaptive Control Example

To illustrate the performance of the direct adaptive controller, we tested it in a computer simulation of a system with inertia matrix given by

$$J = \begin{bmatrix} 1200 & 100 & -200 \\ 100 & 2200 & 300 \\ -200 & 300 & 3100 \end{bmatrix} \text{Kg} \cdot \text{m}^2 \tag{36}$$

We assume the inertia matrix to be completely unknown, and we start with an initial estimate of $\hat{J} = \mathbf{0}$. The goal is to drive the body to be aligned with the inertial reference frame from an initial condition (as used in Eq. 7) given by the Euler angles [1.9168, −0.4876, 1.9168] rad, which corresponds to a quaternion $[q, q_4] = [0.57, 0.57, 0.57, 0.159]$. The desired slewing maneuver then covers an eigenangle of 162 deg. Bounded actuator noise, uniformly distributed in the interval ± 1000, is included in the simulation. This noise is an order of magnitude smaller than the maximum control signals in this application. Gaussian measurement noise is also included with the standard deviation of the quaternion measurements being 0.01 and the standard deviation of the angle rate measurements being 0.001 rad/s. These measurement noises are consistent with those obtained using relatively inexpensive sensors. The trajectories of the controller are shown in Fig. 1, for the quaternion and Fig. 2 for the Euler angles.

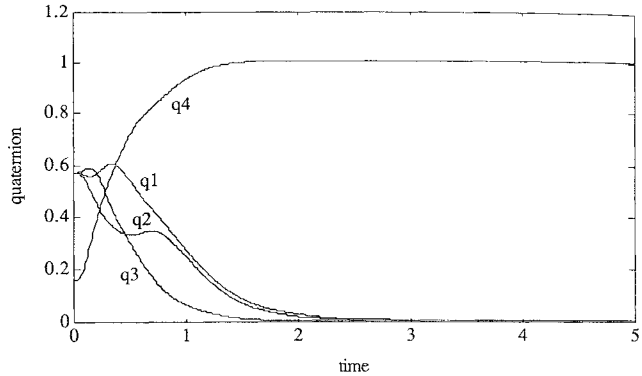

Fig. 1 Quaternion parameters for direct adaptive control example.

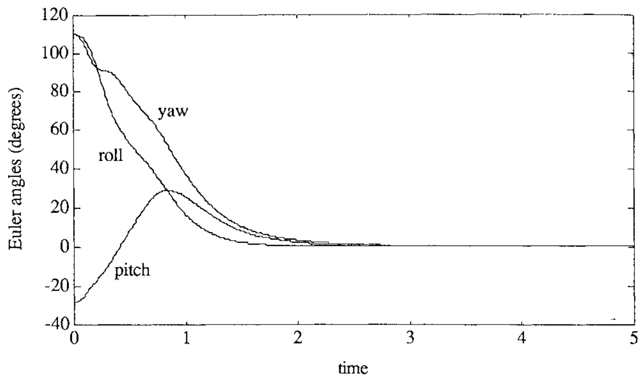

Fig. 2 Euler angles for direct adaptive control example.

B. Indirect Adaptive Control of the Crew/Equipment Retriever

The attitude slewing control system of a spacecraft, like the proposed CER, must operate with large uncertainties in the inertia matrix. The CER is a space-based robot used to retrieve objects detached from the space station. The object to be retrieved, or target, is captured in a net (which opens and closes) and is returned to the space station. The largest target the CER is designed to intercept is an astronaut plus spacesuit and manned maneuvering unit. The mass of this target is comparable to the fully fueled CER. The large relative mass and the long lever arm of an object in the net combine to yield large changes in the inertia matrix. These changes are not known a priori since the target may vary and the location and orientation of the target after capture are uncertain.

The control law (11), using the RLS inertia matrix estimate, was used to slew the CER in computer simulation. The results depicted are for the worst-case scenario where the target is a person equiped with the manned maneuvering unit. In this case the inertia matrices relative to the empty CER (J_0) and the CER with a fully equiped crew member (J_1) are given by (in *slug ft*^{2})

$$J_0 = \begin{bmatrix} 39.6 & 0 & 0 \\ 0 & 55 & 0 \\ 0 & 0 & 55 \end{bmatrix}$$
$$J_1 = \begin{bmatrix} 112.92 & 8.44 & -111.88 \\ 8.44 & 527.14 & -17.00 \\ -111.88 & -17.00 & 497.54 \end{bmatrix} \tag{37}$$

The parameters in the control law (11) were selected based on the simulations as $\alpha = \gamma = 0.22$, in order to result in a reasonable settling time (approximately 50 s) and damping when the inertia matrix is known. The control updating time in the simulations is 0.1 s whereas the updating time for the parameter estimator is 0.3 s. A forgetting factor is included in the RLS algorithm since both measurement and actuator noises are present. The RLS algorithm is then equivalent to a Kalman filter.

The initial orientation and the measurement noises are the same as in the previous example. Bounded actuator noise uniformly dis-

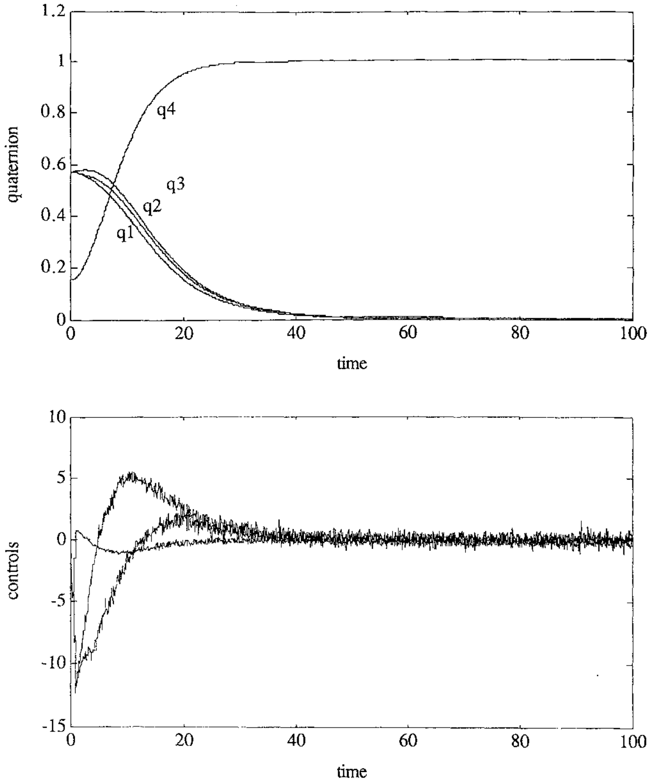

Fig. 3 Adaptive control of CER.

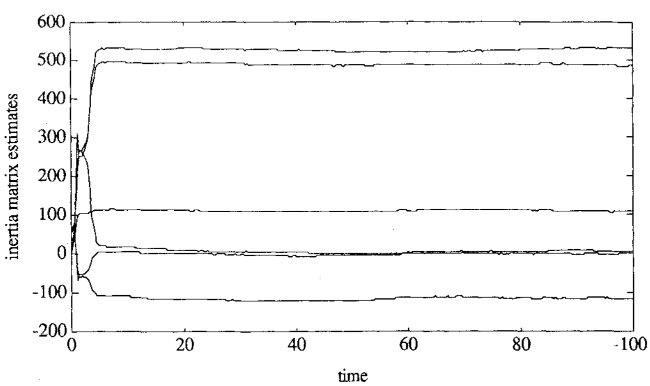

Fig. 4 Inertia matrix parameter estimates.

tributed in the range ± 1 is included in the simulation. This noise is two orders of magnitude smaller than the maximum control signals that represent reasonable actuator performance. The empty CER inertia matrix $\boldsymbol{J}_0$ is used to initialize the inertia matrix estimator. The time histories of the quaternions and control torques are given in Fig. 3. The noise on the control torques initially consists of measurement noise. Once the state trajectory nears the sliding surface, the control torques become more noisy since the measurement noise causes the signum function to rapidly oscillate. This rapid oscillation of the control torque can be removed by filtering the measurements and by addition of a dead zone to the signum function. The estimated inertia matrix parameters are presented in Fig. 4, and the final estimates (after 100 iterations) yield

$$\hat{J}(100) = \begin{bmatrix} 108.7 & 1.8 & -116.3 \\ 1.8 & 529.6 & 3.8 \\ -116.3 & 3.8 & 499.9 \end{bmatrix} \tag{38}$$

which is close to the actual inertial matrix $\boldsymbol{J}_1$ above. Figure 4 shows how rapidly the estimated parameters converge to their final values. Also eigenaxis rotation is indicated by the first three elements of the quaternion remaining proportional to their initial values (all equal in this case). From Fig. 3, we see that the adaptive controller generates near eigenaxis rotations.

VII. Conclusions

An adaptive controller has been developed for rotational maneuvers of a rigid body with considerable dynamic uncertainties. The inertia matrix is supposed to be unknown, which might be the case of a mobile manipulator in space designed to pick up objects of various sizes and weights. Unmodeled dynamic effects are also taken into account in a control scheme reminiscent of sliding-mode techniques. Global stability of the overall adaptive system has been shown analytically and its behavior tested in computer simulations.

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